Period 4, Jan 28, 2024

Product-to-Sum Formulas

$$\sin\alpha\sin\beta = \tfrac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$
$$\cos\alpha\cos\beta = \tfrac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$
$$\sin\alpha\cos\beta = \tfrac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$
$$\cos\alpha\sin\beta = \tfrac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$$

$$\sin 8x \sin 3x = \tfrac{1}{2}[\cos(8x-3x) - \cos(8x+3x)]$$
$$\tfrac{1}{2}[\cos(5x) - \cos 11x]$$

$$\sin 4x \cos x = \tfrac{1}{2}[\sin(4x+x) + \sin(4x-x)]$$
$$\tfrac{1}{2}[\sin 5x + \sin 3x]$$

Express each of the following products as a sum or difference:

a. $\sin 8x \sin 3x$ **b.** $\sin 4x \cos x$.

Sum-to-Product Formulas

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

Express each sum or difference as a product:

a. $\sin 9x + \sin 5x$ **b.** $\cos 4x - \cos 3x$.

$$\sin 9x + \sin 5x = 2\sin\frac{9x+5x}{2}\cos\frac{9x-5x}{2} = 2\sin\frac{14x}{2}\cos\frac{4x}{2}$$

$$= 2\sin 7x\cos 2x$$

$$\cos 4x - \cos 3x = -2\sin\frac{4x+3x}{2}\sin\frac{4x-3x}{2}$$

$$-2\sin\frac{7x}{2}\sin\frac{x}{2}$$

: $\dfrac{\cos 3x - \cos 5x}{\sin 3x + \sin 5x} = \tan x$.

$$\frac{\cos 3x - \cos 5x}{\sin 3x + \sin 5x} = \frac{-\cancel{2}\sin\frac{3x+5x}{2}\sin\frac{3x-5x}{2}}{\cancel{2}\sin\frac{3x+5x}{2}\cdot\cos\frac{3x-5x}{2}}$$

$$\frac{-\sin\frac{8x}{2}\sin\frac{-2x}{2}}{\sin\frac{8x}{2}\cos\frac{-2x}{2}}$$

$$\frac{-\cancel{\sin 4x}\sin(-x)}{\cancel{\sin 4x}\cdot\cos(-x)}$$

$$\frac{-(-\sin x)}{\cos x}$$

$$\frac{\sin x}{\cos x} = \tan x$$

Odd/Even Theorems

$\cos(-x) = \cos x$ Even

$\sin(-x) = -\sin x$ odd

Verify the identity. $\frac{\cos T}{\sin T}$

$\cot t - \sin 2t = \underline{\cot t \cos 2t}$

$\sin 2T = \boxed{2\sin T \cos T}$

$\cos 2T = \cos^2 T - \sin^2 T$

$= 2\cos^2 T - 1$

$= 1 - 2\sin^2 T$

Use the appropriate double-angle formula and rewrite the right side of the identity.

$\cot t\left(1 - 2\sin^2 t\right)$

Distribute the $\cot t$ and rewrite the right side of the equation.

$\cot T(1 - 2\sin^2 T)$

$\cot T - 2\cot T \sin^2 T$

$\cot T - 2\frac{\cos T}{\cancel{\sin T}} \cdot \frac{\sin^{\cancel{2}} T}{1}$

$\cot T - \boxed{2\cos T \sin T}$

$\cot T - \sin 2T$

Use the power-reducing formulas to rewrite the expression as an equivalent expression that does not contain powers of trigonometric functions greater than 1.

$160\sin^2x\cos^2x = 160\left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right) = \frac{160}{4}(1-\cos 2x)(1+\cos 2x)$

$160\sin^2x\cos^2x = 20-20\cos 4x$

$2x=\theta$

$40[1+\cancel{\cos 2x}-\cancel{\cos 2x}-\cos^2 2x]$

$40[1-\cos^2 2x]=40[1-\cos^2\theta]$

$40\left[1-\frac{1+\cos 2\theta}{2}\right]=40\left[\frac{2}{2}-\frac{1+\cos 2\theta}{2}\right]$

$40\left[\frac{2-1-\cos 2\theta}{2}\right]=\frac{40}{2}[1-\cos 2\theta]$

$20[1-\cos 2(2x)]=20-20\cos 4x$

Power-Reducing Formulas

$$\sin^2\theta=\frac{1-\cos 2\theta}{2}\qquad \cos^2\theta=\frac{1+\cos 2\theta}{2}\qquad \tan^2\theta=\frac{1-\cos 2\theta}{1+\cos 2\theta}$$

If $\tan\alpha=\frac{4}{3}$, $0^\circ<\alpha<90^\circ$, then find the exact value of each of the following.

Quad I $(+,+)$

$\frac{0}{2}<\frac{\alpha}{2}<\frac{90}{2}$

a. $\sin\frac{\alpha}{2}$ **b.** $\cos\frac{\alpha}{2}$ **c.** $\tan\frac{\alpha}{2}=\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}$

$$\sin\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{2}}$$
$$\cos\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{2}}$$
$$\tan\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

$0<\alpha<45$ Quad I $(+,+)$

$\tan^2\alpha+1=\sec^2\alpha$

$\left(\frac{4}{3}\right)^2+1=\sec^2\alpha$

$\frac{16}{9}+\frac{9}{9}=\sec^2\alpha$

$\sqrt{\frac{25}{9}}=\sqrt{\frac{1}{\cos^2\alpha}}$

$\pm\frac{5}{3}=\frac{1}{\cos\alpha}$

$\pm\frac{3}{5}=\cos\alpha$

$\sin\frac{\alpha}{2}=\pm\sqrt{\frac{1-\frac{3}{5}}{2}}=+\sqrt{\frac{\frac{5}{5}-\frac{3}{5}}{2}}$

$+\sqrt{\frac{\frac{2}{5}}{\frac{2}{1}}}=\sqrt{\frac{\cancel{2}}{5}\cdot\frac{1}{\cancel{2}}}$

$\sqrt{\frac{1}{5}}=\frac{1}{\sqrt5}\cdot\frac{\sqrt5}{\sqrt5}=\frac{\sqrt5}{5}$

$\cos\frac{\alpha}{2}=\sqrt{\frac{1+\frac{3}{5}}{2}}$

$\sqrt{\frac{\frac{5}{5}+\frac{3}{5}}{2}}$

$\sqrt{\frac{\frac{8}{5}}{\frac{2}{1}}}=\sqrt{\frac{\cancel{8}\,4}{5}\cdot\frac{1}{\cancel{2}}}=\frac{\sqrt4}{\sqrt5}\cdot\frac{\sqrt5}{\sqrt5}=2\sqrt5/5$

a. $\sin\frac{\alpha}{2}=\frac{\sqrt5}{5}$

(Simplify y

b. $\cos\frac{\alpha}{2}=\frac{2\sqrt5}{5}$

(Simplify y

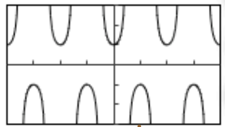
c. $\tan\frac{\alpha}{2}=\frac{\frac{\sqrt5}{5}}{\frac{2\sqrt5}{5}}$

(Simplify y

$\frac{\cancel{\sqrt5}}{\cancel{5}}\cdot\frac{\cancel{5}}{2\cancel{\sqrt5}}=\boxed{\frac{1}{2}}$

To the right, half of an identity and the graph of this half are given. Use the graph to make a conjecture as to what the right side of the identity should be. Then prove your conjecture. The viewing window is $[-2\pi, 2\pi, \pi/2]$ by $[-3, 3, 1]$.

$$\frac{\cot x + \tan x}{\cot x - \tan x} = ?$$



$y = \sec 2x$

$$\frac{\frac{\cos x}{\sin x}\cdot\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}\cdot\frac{\sin x}{\sin x}}{\frac{\cos x}{\sin x}\cdot\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}\cdot\frac{\sin x}{\sin x}}$$

$$\downarrow$$

$$\frac{\frac{\cos^2 x + \sin^2 x}{\sin x \cos x}}{\frac{\cos^2 x - \sin^2 x}{\sin x \cos x}}$$

Which of the following completes the above identity?

- cos (2x)
- − cos (2x)
- sec (2x) ✓
- − sec (2x)

Which of the following proves the conjectured identity?

- $\dfrac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x}$
- $\dfrac{\cos^2 x + \sin^2 x}{\sin^2 x - \cos^2 x}$
- $\dfrac{\cos^2 x + \sin^2 x}{\cos^2 x - \sin^2 x}$ ✓
- $\dfrac{\sin^2 x - \cos^2 x}{\cos^2 x + \sin^2 x}$

$$\frac{(\cos^2 x + \sin^2 x)}{\cancel{\sin x \cos x}} \cdot \frac{\cancel{\sin x \cos x}}{(\cos^2 x - \sin^2 x)}$$

$$\frac{1}{\cos^2 x - \sin^2 x} = \frac{1}{\cos 2x}$$

$y = \sec 2x$

Period

old period

$$\frac{2\pi}{2} = \frac{2x}{2}$$

new period

$\pi = x$ New period

Find an algebraic expression equivalent to the given expression. Hint: form a right triangle.

$\cos\left(2\cos^{-1} x\right)$

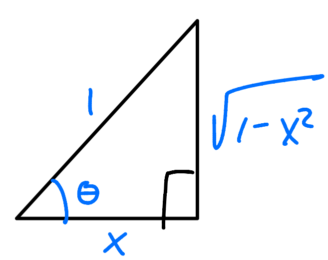
$\cos^{-1} x = \theta$

$\cos\theta = x$

$\cos 2\theta = 2\cos^2\theta - 1$

$= 2x^2 - 1$

$\cos\theta = \frac{x}{1}$



$\cos^2\theta + \sin^2\theta = 1$

$x^2 + \sin^2\theta = 1$

$\sin^2\theta = 1 - x^2$

$\sin\theta = \pm\sqrt{1-x^2}$

$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{b \cdot d}$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

Use the power-reducing formulas to rewrite the expression as an equivalent expression that does not contain powers of trigonometric functions greater than 1.

$4\sin^4 x$

$$= 4(\sin^2 x)^2 = 4 \cdot \sin^2 x \cdot \sin^2 x = 4\left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right)$$

$4\sin^4 x =$ [redacted]

$\cos^2 2x \qquad 2x = \theta$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$= \frac{1+\cos 2(2x)}{2}$$

$$\frac{1+\cos 4x}{2}$$

$$\frac{\cancel{4}}{1} \cdot \frac{(1-\cos 2x)(1-\cos 2x)}{\cancel{4}}$$

$$1 - \cos 2x - \cos 2x + \cos^2 2x$$

$$1 - 2\cos 2x + \cos^2 2x$$

$$1 - 2\cos 2x + \frac{1+\cos 4x}{2}$$

$$\frac{2 \cdot 1 - 2 \cdot 2\cos 2x}{2 \cdot 1} + \frac{1+\cos 4x}{2}$$

$$\frac{3 - 4\cos 2x + \cos 4x}{2}$$

Verify the identity $5 \sin(12t) = 20 \sin 3t \cos^3 3t - 20 \sin^3 3t \cos 3t$.

$\sin 2\theta = 2\sin\theta\cos\theta$

$2\theta = 12T \Rightarrow \theta = 6T$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

Which type of identities will be the most useful for changing the argument of the function on the left side by a factor of $\frac{1}{2}$? Select all that apply.

- [x] A. Double-angle formulas
- [ ] B. Quotient identities
- [ ] C. Even-odd identities
- [ ] D. Pythagorean identities

$5\sin(2(6T)) =$

$5 \cdot 2\sin 6T \cos 6T$

$10 \sin 2(3T) \cos(2 \cdot 3T)$

$10(2 \sin 3T \cos 3T)(\cos 2(3T))$

$20 \sin 3T \cos 3T [\cos^2 3T - \sin^2 3T]$

Rewrite the left side using one of the appropriate formulas once.

5 sin (12t) = 10 sin (6t) cos (6t)

What formulas can then be applied to rewrite the expression in the previous step as $20 \sin 3t \cos^3 3t - 20 \sin^3 3t \cos 3t$? Select all that apply.

$20 \cos^3 3T \cdot \sin 3T - 20\sin^3 3T \cos 3T$

- [x] A. $\sin 2\theta = 2\sin\theta\cos\theta$
- [x] B. $\cos 2\theta = \cos^2\theta - \sin^2\theta$
- [x] C. $\sin^2\theta + \cos^2\theta = 1$
- [ ] D. $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$
- [ ] E. $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
- [ ] F. $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
- [ ] G. $\cos(-x) = \cos x$
- [ ] H. $\tan x = \dfrac{\sin x}{\cos x}$
- [ ] I. $\sin(-x) = -\sin x$

Double-Angle Formulas

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$